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INSTRUCTIONAL GUIDE FOR ALGEBRA 1, GRADES 9 TO 12.

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REPORT NUMBER X-65 PUB DATE 66
EDRS PRICE MF-$0.25 HC-$2.20 53P.

DESCRIPTORS- *ALGEBRA, *CURRICULUM GUIDES, *SECONDARY SCHOOL MATHEMATICS, *TEACHING GUIDES, COURSE CONTENT, INSTRUCTIONAL MATERIALS, MATHEMATICS, TEACHING PROCEDURES, LOS ANGELES, CALIFORNIA,

THIS INSTRUCTIONAL GUIDE WAS WRITTEN TO PROVIDE ASSISTANCE TO TEACHERS IN DEVELOPING THE BASIC CONCEPTS AND SKILLS OF ELEMENTARY ALGEBRA. THE CONTENT FOR EACH UNIT INCLUDES GOALS, A SEQUENTIAL DEVELOPMENT OF THE UNIT, AND SPECIFIC TEACHING SUGGESTIONS. THE TABLE OF CONTENTS FOR THE COURSE IS A DUPLICATION OF THE TABLE IN DOLCIANI, BERMAN, AND FREILICH'S "MODERN ALGEBRA, STRUCTURE AND METHOD," BOOK 1. AN ALTERNATE SEQUENCE USING THE TEXT, KEEDY, JAMESON, AND JOHNSON'S "EXPLORING MODERN MATHEMATICS," BOOK 3 ELEMENTARY ALGEBRA, IS PROVIDED AND RECOMMENDED FOR USE WITH HIGH-ABILITY GROUPS TO CULMINATE THE ON-GOING SEQUENCE PRESENTED IN BOOKS 1 AND 2. (RP)

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# Instructional Guide for

# ALGEBRA 1

## GRADES 9 TO 12

LOS ANGELES CITY SCHOOLS
Division of Instructional Services
Curriculum Branch
Publication No. X-65
1966

# FOREWORD

Algebra 1 and 2 comprise a modern first-year course in elementary algebra which is basic to the study of the physical sciences, as well as to all advanced topics in mathematics, statistics, probability, technology, and related fields. First-year algebra usually is followed by a one-year course in geometry.

This instructional guide has been written to provide assistance and some guidance to teachers in developing the basic concepts and skills of elementary algebra. The content for each unit includes goals, sequential development of the unit, and specific teaching suggestions.

Emphasis throughout is on clarifying and simplifying the development of modern concepts presented in the texts. Explanatory material is included in the guide to provide background information. The teacher is expected to use his judgment as to what depth of development of each topic is appropriate for a particular class.

## ACKNOWLEDGMENTS

Grateful acknowledgment of their assistance is made to the many persons who have contributed directly or indirectly to the preparation of the Instructional Guide for Algebra 1.

Teachers have assisted in the experimental use of texts, have served as members of an algebra textbook selection committee, and have helped develop a brief outline for use as the new texts were introduced.

More recently, members of the Project Advisory Committee for Algebra 1 and 2, read and evaluated the manuscript for this publication, offered constructive criticisms, added helpful teaching suggestions, and participated in discussions leading to the solutions of problems related to the work. Gratitude is expressed to the following persons who served as members of this committee:

| | |
|---|---|
| John S. Fox | Northridge Junior High School |
| Alvin Harrison | Mann Junior High School |
| Charles Helsel | Burbank Junior High School |
| Evelyn Hovey | Marshall High School |
| Alex Morrison | Francis Polytechnic High School |
| Michael Pomerantz | Venice High School |
| Theodore C. Rambach | Banning High School |
| Winifred Rhodes | Markham Junior High School |
| Elwyn A. Wherry, Principal | Northridge Junior High School |

Appreciation is expressed to Keith F. Smith, Supervisor of Mathematics, Division of Secondary Education, for working with the production committee and reviewing the manuscript.

Particular gratitude is conveyed to LEONARD M. WARREN, Consultant, Curriculum Branch. The staff is indebted to him for his excellent contributions in developing, organizing, and preparing the manuscript. Mr. Warren's rich background in mathematics, his particpation in the Maryland Project on an academic grant, his varied teaching experiences, and his special talent for writing fitted him well for the task of writing this guide.

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# TABLE OF CONTENTS


Page

FOREWORD . . . . . iii

ACKNOWLEDGMENTS . . . . . iv

OBJECTIVES OF ALGEBRA 1 and 2 . . . . . vi

TOPICAL OUTLINES FOR ALGEBRA 1 and 2 (Dolciani) . . . . . vii

TOPICAL OUTLINES FOR ALGEBRA 1 and 2 (Keedy) . . . . . viii

ALGEBRA 1 (Dolciani) . . . . . 1

UNIT I: SYMBOLS AND SETS . . . . . 3

UNIT II: VARIABLES AND OPEN SENTENCES . . . . . 7

UNIT III: AXIOMS, EQUATIONS, AND PROBLEM SOLVING . . . . . 11

UNIT IV: THE NEGATIVE NUMBERS . . . . . 17

UNIT V: EQUATIONS, INEQUALITIES, AND PROBLEM SOLVING . . . . . 23

UNIT VI: WORKING WITH POLYNOMIALS . . . . . 29

UNIT VII: SPECIAL PRODUCTS AND FACTORING . . . . . 35

ALGEBRA 1 (Keedy) . . . . . 45

TO THE TEACHER . . . . . 46

UNIT I: APPLIED PROBLEMS, CONJUNCTIONS OF EQUATIONS . . . . . 47

UNIT II: SIMILAR FIGURES . . . . . 47

UNIT III: POLYNOMIALS IN SEVERAL VARIABLES . . . . . 48

UNIT IV: NUMBER SENTENCES AND PROOFS . . . . . 48

UNIT V: THE SYSTEM OF REAL NUMBERS . . . . . 49

# OBJECTIVES OF ALGEBRA 1 and 2

This one-year course in algebra is designed to prepare pupils to function more effectively in a society which is increasingly more dependent upon manpower trained in mathematics. Instruction provides pupils with some insight into the basic principles that underlie the structure of mathematics. Manipulative skills and the techniques to be learned are developed as reflections of this structure and as logical consequences of the principles which characterize the real number system.

Pupils who study the material covered in this course should:

1. Acquire facility in applying algebraic concepts and skills.

2. Understand the principles on which the algebraic processes are based.

3. Increase their comprehension of the nature of our number system and of the properties of operations with the elements of various algebraic systems.

4. Organize information in coherent logical sequence, identify and represent variables algebraically, and translate the conditions imposed upon these variables into mathematical sentences.

5. Develop their problem-solving ability.

6. Understand that there is more than one fixed and prescribed way of solving a problem.

7. Improve their competence at estimating answers and judging if a solution is reasonable.

8. Make certain generalizations as a result of inductive discovery.

9. Develop skill in devising algebraic proofs, as well as understanding of the role of deductive proof in mathematics.

10. Acquire respect and regard for accuracy and learn to check work conscientiously and carefully in order to achieve greater efficiency.

# TOPICAL OUTLINES FOR ALGEBRA 1 and 2

## Algebra 1

Text: Dolciani, Berman, Freilich: Modern Algebra, Structure and Method Book One

| UNIT | | Number of Teaching Days |
|---|---|---|
| I: | SYMBOLS AND SETS | 6 |
| II: | VARIABLES AND OPEN SENTENCES | 9 |
| III: | AXIOMS, EQUATIONS, AND PROBLEM SOLVING | 12 |
| IV: | THE NEGATIVE NUMBERS | 10 |
| V: | EQUATIONS, INEQUALITIES, AND PROBLEM SOLVING | 12 |
| VI: | WORKING WITH POLYNOMIALS | 14 |
| VII: | SPECIAL PRODUCTS AND FACTORING | 17 |

## Algebra 2

Text: Dolciani, Berman, Freilich: Modern Algebra, Structure and Method Book One

| UNIT | | Number of Teaching Days |
|---|---|---|
| VIII: | WORKING WITH FRACTIONS | 20 |
| IX: | GRAPHS | 10 |
| X: | SENTENCES IN TWO VARIABLES | 15 |
| XI: | THE REAL NUMBERS | 13 |
| XII: | QUADRATIC EQUATIONS AND INEQUALITIES | 15 |
| XIII: | FUNCTIONS AND VARIATIONS | 7 |
| XIV: | COMPREHENSIVE REVIEW AND TESTS | (5) |

# TOPICAL OUTLINES FOR ALGEBRA 1 and 2

## Algebra 1

Text: Keedy, Jameson, Johnson: Exploring Modern Mathematics, Book 3, Elementary Algebra

| UNIT | | Number of Teaching Days |
|---|---|---|
| I: | APPLIED PROBLEMS, CONJUNCTIONS OF EQUATIONS | 25 |
| II: | SIMILAR FIGURES | 20 |
| III: | POLYNOMIALS IN SEVERAL VARIABLES | 17 |
| IV: | NUMBER SENTENCES AND PROOFS | 20 |

## Algebra 2

Text: Keedy, Jameson, Johnson: Exploring Modern Mathematics, Book 3, Elementary Algebra

| UNIT | | Number of Teaching Days |
|---|---|---|
| V: | THE SYSTEM OF REAL NUMBERS | 25 |
| VI: | FRACTIONAL PHRASES AND EQUATIONS | 20 |
| VII: | RADICAL NOTATION AND NUMBER SENTENCES | 20 |
| VIII: | GRAPHS, RELATIONS AND FUNCTIONS | 17 |

# A L G E B R A 1

Text: Dolciani, Berman, Freilich:
*Modern Algebra, Structure and Method*
*Book One*

# UNIT I: SYMBOLS AND SETS

## GOALS

Pupils should be able to demonstrate that they have learned to:

1. Represent numbers on a line.
2. Compare numbers, using the signs of equality and inequality.
3. Understand the meaning of membership in a set and recognize different kinds of sets.
4. Draw the graph of a set.
5. Interpret the symbols of inclusion.
6. Perform indicated operations in the proper order.

## SEQUENTIAL DEVELOPMENT OF THE UNIT

### Representation of Numbers on a Line

The elements of the real number system may be paired with the points on a line. This technique of depicting numbers on a number line arranges the numbers in order of magnitude and illustrates that each number is associated with a unique point on the line.

The number that is coupled with a point on the number line is called the coordinate of that point; the drawing which shows the position of the number--that is, the point which is matched with the number--is the graph of that number.

Numerals corresponding to points on the number line are determined.

### Comparison of Numbers

Symbols of equality (=) and inequality ($\neq$ , $>$ , $<$) are used to indicate the relationship between expressions.

Statements are judged to be either true or false, or are made true by replacing question marks with appropriate symbols.

### Sets of Numbers

The word set is used to denote any well-defined collection of objects, things, or symbols; in mathematics, sets of so-called mathematical objects are those usually considered: sets of numbers, sets of points, sets of lines, sets of angles.

Each object, symbol, person, or thing in a particular set is an element or member of that set. A set may have an unlimited number of elements, a finite number of elements, or no elements. A set that contains no elements is called the empty set, or null set, and is indicated by the symbol $\emptyset$.

## UNIT I: SYMBOLS AND SETS

### SEQUENTIAL DEVELOPMENT OF THE UNIT

Sets of Numbers (contd)

The elements of a set are specified by listing the members (the roster method), by describing the unique conditions for membership (the rule method), or by locating the members on the number line (the graphic method).

Order of Operations

Symbols of inclusion (grouping symbols) are employed to show the order of operations. If such punctuation is omitted, the multiplications and divisions are performed first in the order in which they occur. The additions and subtractions are then accomplished in the order in which they appear.

Expressions are simplified in which punctuation is supplied or convention is observed.

### TEACHING SUGGESTIONS

The teacher may find it helpful to:

1. Recognize the existence of the negative numbers and extend the number line to include the negative real numbers.
2. Explain that a coordinate is one of a set of numbers which locate a point in space. If the point is known to be on a given line, only one coordinate is needed; if in a plane, two are required; if in space, three. The set of points corresponding to a set of numbers is called the graph of the set.
3. Review the signs of inequality ($>$ , $<$ , $\neq$) and introduce the symbols $\geq$ and $\leq$ .
4. Instruct pupils to replace question marks with appropriate symbols which will make the resulting statements true.
5. Introduce the concepts, the vocabulary, and some of the symbolism of sets to stimulate interest and promote understanding. However, abstract set theory is not a part of the course.
6. Utilize the notion of set systematically during the course to help clarify other mathematical ideas and unify several apparently different topics in mathematics. Do not, however, seek to have pupils learn this one thing to perfection before going on to the next section. Emphasize concepts, rather than vocabulary.
7. Establish concrete or intuitive notions before presenting abstract concepts. Pupils often have dealt with collections of objects--for example, collections of stamps, autographs, or post cards. In each case, the pupil is able to tell whether an object belongs to his collection. Such collections are called sets.

   Thus, a set is any well-defined collection of objects, physical or conceptual; "well-defined" is interpreted to mean that it always can be determined whether an object belongs to the set. This concept can be extended and strengthened as different sets of numbers are examined.

TEACHING SUGGESTIONS (contd)

8. Explain that the symbols of inclusion indicate that the expressions enclosed are to be regarded as a single quantity.

9. Illustrate the need for establishing a universally accepted order for performing operations with algebraic expressions. For example:

   $16 - 8 \div 4$ If the division is done first, $16 - 2 = 14$; but if the subtraction is accomplished first, $8 \div 4 = 2$.

   Certain conventions must be followed, and some order of performing the indicated operations must be agreed upon if mathematicians are to communicate with one another.

10. Provide additional examples, with more able classes, of expressions which contain grouping symbols enclosed by other grouping symbols. Consider, in greater detail, problems of this nature:

$$3\left\{\left[\underline{4} \times (7 - \underline{2})\right] + 6\right\}$$

   Emphasize that symbols are removed one set at a time, beginning with the innermost set.

11. Distinguish between the meaning of four times two plus five and four times the <u>sum</u> of two and five. Relate the use of parentheses to the accurate translation of such expressions in the next unit.

12. Give less emphasis to, or even, in some classes, omit the use of the bar as a symbol of inclusion. Instead, instruct pupils to rewrite the exercises, replacing the bar with parentheses. For example:

   $5\ \overline{6 + 8}$ often is misinterpreted; $5(6 + 8)$ is much clearer.

   It may be best also to avoid the use of braces because they may be used in two ways:

   a. as grouping symbols
      or
   b. to enclose the elements of a set

EVALUATION

An evaluation program includes not only the checking of completed work at convenient intervals but also continuous appraisal. Evaluation is designed to improve instruction by determining the effectiveness of the techniques, materials, and content. It serves also to determine pupils' readiness for instruction, to diagnose weaknesses and strengths, and to provide reinforcement.

A basic principle of evaluation of mathematics achievement is measurement in terms of the objectives of the instruction. The goals for each unit, including concepts , principles, understandings, and skills are clearly defined at the beginning of the unit.

EVALUATION (contd)

Written tests, homework assignments, chalkboard work, and quizzes may be used to measure the degree to which pupils have achieved the goals for the unit. Available, also, are the Chapter Tests at the conclusion of each chapter in the textbook and the examinations included in McLean: Progress Tests to Accompany Modern Algebra, Structure and Method, Book 1.

Oral activities which teachers may use include:

Pupil explanations of approaches used in new situations.

Pupil justifications of statements.

Pupil restatement of problems.

Oral quizzes.

Reports.

# UNIT II: VARIABLES AND OPEN SENTENCES

## GOALS

Pupils should be able to demonstrate that they have learned to:

1. Evaluate algebraic expressions containing variables.
2. Identify factors, coefficients, and exponents.
3. Solve open sentences.
4. Interpret algebraic expressions.
5. Translate verbal phrases into algebraic expressions.
6. Write open sentences.

## SEQUENTIAL DEVELOPMENT OF THE UNIT

### Algebraic Expressions Containing Variables

A variable is defined as a symbol which may represent any element of a given set.

Replacement sets, the domain of the variable, and expressions are illustrated. For an expression which is written as the sum of several quantities, each of these quantities is called a term of the expression. Algebraic expressions are evaluated by replacing the variables in the expressions with the numerals which the variables represent.

### Factors, Coefficients, and Exponents

An exponent indicates how many times an expression (called the base) is to be used as a factor. The product is the power; for example, $3^2$, or 9, is the second power of three.

Expressions containing exponents are evaluated.

### Open Sentences

An open mathematical sentence, such as $2x + 5 = 19$ or $2x + 5 < 19$, is neither true nor false, but becomes either true or false depending upon the replacement made for the variable, x, from an appropriate list (or set) of numbers (the domain of the variable). Any number that makes the sentence true is said to satisfy the sentence; the set of all numbers which satisfies the sentence is called the solution set of the sentence.

Selections are made from the replacement set to determine whether the resulting sentence is true or false.

## SEQUENTIAL DEVELOPMENT OF THE UNIT (contd)

### Solution of Problems

The process of solving problems involves: (1) identifying the variable and representing it algebraically, (2) translating the conditions imposed on that variable into a mathematical sentence, (3) solving that sentence, and (4) determining whether that solution is acceptable and reasonable.

Simple algebraic expressions are interpreted, verbal phrases are converted into algebraic expressions, and open sentences are written.

## TEACHING SUGGESTIONS

The teacher may find it helpful to:

1. Illustrate the meaning of ideas, such as open sentence, domain of the variable, and solution set, with the following example:

   Consider the following multiple-choice test item:

   _____ was the thirty-second President of the United States.

   a. Woodrow Wilson

   b. Calvin Coolidge

   c. Franklin Roosevelt

   d. Dwight Eisenhower

   The expression, "_____ was the thirty-second President of the United States," is neither true nor false; it is an open sentence. The variable is the blank, a symbol which holds the place for or represents the choices listed. The list of names is the domain of the variable (or replacement set); substitutions for the blank are limited to the names which appear in this set. The open sentence becomes either true or false depending upon replacements made for the variable from the list of names (the domain of the variable). Any name that makes the sentence true satisfies the sentence. The set of all names which satisfy the sentence is the solution set of the sentence.

2. Stress that a variable is a symbol which represents a member or element of a designated set. It does not represent an unknown entity.

3. Emphasize that every member of a solution set makes the statement true.

4. Distinguish between terms and factors. The expression abc, which means the product of a, b, and c, consists of one term but has several factors.

5. Explain that, although $3x + 5y$ is an algebraic expression of two terms, the expression $\frac{3x + 5y}{8}$ is a single term, representing one quotient.

TEACHING SUGGESTIONS (contd)

6. Provide additional drill and practice in evaluating algebraic expressions. Assign exercises, of the type $x^2 + y^2 + z^2$, replacing x, y, and z with numbers such as 5, 2, and 3, respectively.
7. Review the symbols $\geq$ and $\leq$ before the exercises requiring a knowledge of these are assigned.
8. Require pupils to replace the variable in the open sentences with the elements of the replacement set to determine whether the resulting statement is true or false.
9. Request pupils to graph several solution sets.
10. Postpone development of formal methods of solving equations and inequalities. Instead, permit pupils to devise their own techniques.
11. Encourage pupils to interpret simple algebraic expressions by describing a physical situation that can be illustrated by the expression.
12. Stress the accurate translation of verbal phrases into algebraic expressions, and provide exercises to supplement those presented in the text.
13. Emphasize the need for identifying what it is one is asked to find in a problem and for representing it with an appropriate variable.
14. Place emphasis on writing open sentences.
15. Insist that pupils check the solution of a problem by testing to see whether it satisfies the conditions described by the problem, rather than simply by substituting it in the original equation or any of the derived equations.
16. Remind pupils to read instructions carefully before beginning an assignment.

EVALUATION

See the evaluation section for Unit I, pages 5 and 6 of this guide.

# UNIT III: AXIOMS, EQUATIONS, AND PROBLEM SOLVING

## GOALS

Pupils should be able to demonstrate that they have learned to:

1. Identify the properties of an equivalence relation.
2. Distinguish certain characteristics of operations with the elements of the non-negative number system.
3. Compute more accurately and more swiftly by using the properties of addition and multiplication.
4. Simplify expressions by combining similar terms.
5. Transform equations into equivalent equations.
6. Determine the solution sets of equations.

## SEQUENTIAL DEVELOPMENT OF THE UNIT

### Properties of the Equals Relation

The reflexive, symmetric, and transitive properties are characteristics of an equality. They are assumed to be true and may be used to deduce the consequences of operating with equalities.

Illustrations of the properties of equality are presented, and the "substitution principle" is established. This principle states that if two quantities are equal, one may be replaced by the other in any expression or in any statement without destroying the validity of the statement.

### Properties of Addition and Multiplication

A mathematical system is said to be <u>closed</u> under a specified binary operation if the result of the operation on every ordered pair of elements is a unique element of the system. The set of whole numbers is closed under addition, for example, if the sum of any two whole numbers is a <u>unique</u> whole number.

The result of adding or multiplying two numbers is independent of the <u>order</u> in which the two numbers are combined. The <u>commutative</u> property of addition and multiplication expresses the principle that the sum or product is not altered by the <u>order</u> in which the two elements are added or multiplied.

Addition and multiplication are <u>associative</u> because the result of adding or multiplying several terms is the same, irrespective of the way the elements are <u>grouped</u>.

Multiplication is <u>distributive</u> relative to addition, because the product of a number and the sum of a set of quantities is <u>equivalent</u> to multiplying each member of the set by that number and then combining the result.

## SEQUENTIAL DEVELOPMENT OF THE UNIT

### Properties of Addition and Multiplication (contd)

Zero is the identity element for addition; if zero is added to a given number, that number is unchanged. One is the identity element for multiplication; if any given number is multiplied by one, that number is unchanged. The product of any number and zero is zero.

The properties are cited to justify steps in simplifying expressions and combining similar terms.

### Transformation of Equations

An equation can be transformed into an equivalent equation by application of the properties of equality. If the same number is added to or subtracted from both members of an equation or if both members of an equation are multiplied or divided by the same number, other than zero, the derived equation is equivalent to the original equation. Equations are equivalent if the number which satisfies one equation is a root also of the derived equation.

The process of solving an equation is one of successive applications of the properties of equality, transforming the original equation finally into one that more clearly reveals the number which, when used as a replacement for the variable, will satisfy the equation.

The equality properties are derived, and a technique is developed to determine the solution set of an equation.

## TEACHING SUGGESTIONS

The teacher may find it helpful to:

1. Illustrate the meaning of the reflexive, symmetric, and transitive properties of an equality using sentences written in the following pattern:

_____( statement of a particular relation )_____

   a. Example: _____is as clever as _____.

      If you replace each blank with the same name -- for example, "Jerry is as clever as Jerry," -- the statement obviously is correct. Similar replacement could be made with relations such as "is as heavy as," "looks like," and "runs as swiftly as," and the statement would be correct.

      "Seven is equal to seven" is another example; such relations which are correct when the same term is used as a replacement for both blanks have the reflexive property. Examples of relations that are non-reflexive should be presented: " is smarter than," "is older than," "is less than," "is greater than."

TEACHING SUGGESTIONS

1. (contd)

b. Below are two pairs of sentences. In the first sentence of each pair, replace the first blank with John and the second blank with Katherine. In the second sentence, reverse the position of the names.

John is the cousin of Katherine.
Katherine is the cousin of John.
} Both sentemces are correct.

John is taller than Katherine.
Katherine is taller than John.
} If the first relation is correct, the second sentence is not correct.

Similar examples can be examined with relations such as "is the father of," "is the brother of" (for the set of boys), and "runs faster than" to determine if the relation is correct for the second of the two sentences, assuming that it is correct for the first.

"9 + 6 is equal to 15, and 15 is equal to 9 + 6." Such relations as this, which are correct when the terms are interchanged, possess symmetry. Those relations for which the terms cannot be interchanged are non-symmetric.

c. "If Tom is taller than Steve and Steve is taller than Dave, then Tom is taller than Dave." This sentence is correct.

"If Carl is the father of Lloyd and Lloyd is the father of Dana, then Carl is the father of Dana." This sentence is not correct.

"If 9 + 7 is equal to 16 and 16 is equal to 8 x 2, then 9 + 7 is equal to 8 x 2." This statement is correct; those relations for which this pattern applies are transitive, and those for which it does not apply are non-transitive.

d. Relations which have all three properties are called equivalence relations. The relation is equal to is an equivalence relation. The relation is greater than is non-reflexive and non-symmetric, but transitive.

TEACHING SUGGESTIONS (contd)

2. Demonstrate the effectiveness of employing the properties of addition and multiplication in simplifying calculations.

a. $8 \times 173 \times 125 = 8 \times (173 \times 125)$ associative property
$= 8 \times (125 \times 173)$ commutative property
$= (8 \times 125) \times 173$ associative property
$= 1000 \times 173 = 173{,}000$

b. $82 \times 99 + 18 \times 99 = (82 + 18) \times 99$ distributive property
$= 100 \times 99 = 9900$

c. $9\frac{2}{3} \times 6 = (9 + \frac{2}{3}) \times 6 = (9 \times 6) + (\frac{2}{3} \times 6)$ distributive property
$= 54 + 4 = 58$

What is significant is not the names of these characteristics but how they enable the pupil to be more accurate and efficient.

3. Extend the definition of similar terms to include not only those terms whose varible factors are the same, but also those terms which contain the same power (or powers) of the variable.

4. Emphasize the application of the distributive property to write equivalent expressions when combining terms.

a. $11x + 31x = (11 + 31)x = 42x$

b. $5x + 7y + 3x + 8y = 5x + (7y + 3x) + 8y$
$= 5x + (3x + 7y) + 8y$
$= (5x + 3x) + (7y + 8y)$
$= (5 + 3)x + (7 + 8)y = 8x + 15y$

c. $7ax + 4ax = ax(7 + 4)$
$= ax(11) = 11ax$

5. Broaden the interpretation of the distributive property to show that the product of a monomial and a polynomial is equal to the sum of the product of the monomial and each term of the polynomial.

For example, $2(x + 2y + 3) = 2x + 4y + 6$.

6. Caution pupils that multiplication is not distributive with respect to multiplication; that is:

$2 \times 3 \times 5 \neq (2 \times 3) \times (2 \times 5)$ and

$2[7(2x + 5y)] \neq 14(4x + 10y)$

TEACHING SUGGESTIONS (contd)

7. Point out that the addition, subtraction, division, and multiplication properties of equality are logical and valid consequences of previous assumptions, definitions, and properties. Refer to "An Introduction to Sets and the Structure of Algebra," by Krickenberger and Pearson, for the development of proofs, such as:

   If a, b, and c are elements of the set of numbers and a = b, then a x c = b x c.

   | Proof: | | |
   |---|---|---|
   | | a x c is a number | closure property of multiplication |
   | | a x c = a x c | reflexive property of equality |
   | | a = b | given |
   | | a x c = b x c | substitution principle |

8. Stress that each step in the process of transforming an equation produces an equivalent equation. The number which satisfies each of the derived equations is also the root of the original equation.

9. Postpone the assigning of equations having the variable in both members and the set of problems which follows until after negative numbers have been introduced, thus avoiding situations such as $-4x = -12$. Furthermore, after negative numbers have been considered in greater detail and the "additive inverse" has been reviewed, transformations need no longer be accomplished by the subtraction property of equality.

10. Develop the strategy for solving equations, as follows:

    a. Perform the indicated operations and combine similar terms in each member of the equation.

    b. Transform the equation into equivalent equations by repeated applications of the addition and subtraction properties of equality, attempting to isolate all the terms containing the variable in one member of the equation and the constants (or "knowns") in the other. Sometimes, it may be more desirable to apply the multiplication property of equality first; for example, in equations such as:

    $$\frac{3x}{4} + \frac{x + 9}{5} = 36$$

    c. Apply the multiplication or division property of equality to derive an equivalent equation which reveals the solution set.

TEACHING SUGGESTIONS (contd)

11. Instruct pupils to check the solution set in the <u>given</u> equation, rather than in any of the derived equations.

12. Anticipate the difficulties pupils will encounter with the word problems. Stress the need for identifying the variable and translating accurately. Point out, too, how to represent algebraically two numbers when their sum is known, before assigning tasks involving that relationship.

EVALUATION

See the evaluation section for Unit I, pages 5 and 6 of this guide.

# UNIT IV: THE NEGATIVE NUMBERS

## GOALS

Pupils should be able to demonstrate that they have learned to:

1. Locate and represent the directed numbers on the number line.
2. Order the set of rational numbers according to size.
3. Determine the opposite or additive inverse and the reciprocal of each element of the system.
4. Add, subtract, multiply, and divide directed numbers.

## SEQUENTIAL DEVELOPMENT OF THE UNIT

### Directed numbers and Absolute Value

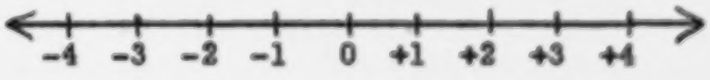
The set of directed numbers includes numbers having signs, positive or negative, indicating that the negative numbers are to be measured, geometrically, in the direction opposite to that in which the positive numbers are measured.

The absolute value of a number is its value without regard for sign; it is the magnitude of the number, or the numerical value of the distance between zero and the number.

The number line is extended to include those numbers less than zero, and the negative numbers are introduced as the coordinates of the points on the line to which they correspond. Graphical representations are made, and applications of positives and negatives to denote opposites (east and west, right and left, gain and loss) and to express familiar situations (increases and decreases, withdrawals and deposits, distances above and below sea level) are considered.

### Order Relation

The directed numbers are arranged in an order. If the number line is pictured horizontally, the larger of two directed numbers is to the right of the smaller.




For example, $-4 < -1$, $2 > -3$, $-1 < 0$. If any real number is less than another, it is located to the left of the second number on a horizontal number line.

Question marks are replaced with inequality symbols to make sentences true, statements are judged to be either true or false, and selections are made from a replacement set to determine the solution set of open sentences.

## SEQUENTIAL DEVELOPMENT OF THE UNIT (contd)

### Additive Inverse

For every element in the set of directed numbers, there is a unique element such that the sum of the two numbers is zero (the identity element for addition). If the sum of two numbers is zero, each is called the additive inverse (or the "opposite," or the "negative") of the other.

The additive inverse of any real number $n$ is defined as the number $x$ such that $n + x = 0$. Since a real number $n$ has only one additive inverse, $x$ is denoted by $-n$, thus $n + (-n) = 0$. Every positive number, $+n$, has an opposite, $-n$, such that their sum $(+n) + (-n) = 0$.

Corresponding pairs of numbers whose sums are zero are examined, the additive inverse of a sum of several numbers is illustrated, and additive inverses of directed numbers and sums of directed numbers are named.

### Operations With Directed Numbers

The rules governing the operations with the rational numbers and the properties possessed by these operations should agree with the same principles which regulate the operations with and the characteristics of the set of positive rational numbers and zero.

With the number system extended to include the negative rational numbers, and the properties of an inverse for addition and closure for subtraction having been gained, the operations with directed numbers may be developed.

To derive the rules for operating with directed numbers, experiments on the number line are conducted ; vectors are added; practical illustrations are cited; patterns may be examined; and, with pupils of greater mathematical maturity, structure may be considered and informal proofs prepared.

## TEACHING SUGGESTIONS

The teacher may find it helpful to:

1. Introduce the study of directed numbers by encouraging pupils to identify positive and negative numbers with consideration of situations, such as an overdrawn bank account, a temperature below zero, an increase in population, or a loss of 10 yards in a football game. The pupils' acquaintance with these numbers is in terms of their use, rather than of their basic properties as numbers. (The negative numbers were not invented to provide closure for subtraction or an inverse for addition. One of the earliest attempts to interpret negative numbers was made by Fibonacci, an Italian of the thirteenth century, who decided that, in determining profit, a negative result implied a loss.) The basic idea underlying this extension of the number system is direction.

TEACHING SUGGESTIONS (contd)

2. Indicate that the negative numbers supply solutions to equations such as $x + 17 = 6$, and provide answers to subtraction problems such as $9 - 13$. This makes subtraction always pos ible and closed (for all rational numbers a and b, $a - b$ is a unique rational number). Negative numbers also serve as roots to equations such as $x + 8 = 0$, thus furnishing an inverse element for addition.

3. Select two points on the number line, P and P', which correspond to +7 and -7, and stress that both points are seven units from the origin, one to the right and one to the left. The number of units, without regard to direction, between the origin (zero) and any point on the line is called the absolute value of the number associated with that point. The absolute value is always a non-negative directed number.

4. Illustrate that the description of one directed number being less than another agrees with the ways in which directed quantities are compared in more familiar situations. For example, if positive numbers reflect profits in business, and negative numbers represent losses, an outcome of +$35 is better than one of -$50; and a result of -$20, although not good, still is better than one of -$40. Similarly, -5° is warmer than -10°.

5. Emphasize that -a, the additive inverse (or "opposite") of "a", does not necessarily represent a negative number. If "a" represents a negative number, for example, -3, then "-a" represents a positive number; that is, $-(-3) = +3$.

   Since $(-3) + (+3) = 0$ and $(-3) + [-(-3)] = 0$, it can be shown that $-(-3) = +3$.

6. Present the material on absolute value and additive inverse before considering addition on the number line in detail. Use the notion of "displacements" as one of the means for developing the rules for addition of directed numbers.

7. Postpone assignment of those exercises which inquire about inverses of sums and absolute values of sums until after the rules for adding directed numbers are established. Return to these questions later to review the concepts.

8. Supplement the section on absolute value with additional drill and practice, especially with expressions containing a variable.

9. Explain that $|x - 5| = 7$ is interpreted to mean that x is located on the number line so that its distance from 5 in either direction is 7.

10. Employ a variety of techniques to develop the rules for addition, such as exercises on the number line, using line vectors; experiments with gains and losses or with increases and decreases; a study of pattern; and examination of mathematical structure.

TEACHING SUGGESTIONS (contd)

11. Clarify the rules for addition, guiding pupils to conclude as a result of their observations, that: the sum of two positive numbers is a positive number; the sum of two negative numbers is a negative number (in each case, the absolute value of the sum is determined by adding the absolute values of the two numbers); the sum of a positive number and a negative number is positive if the positive number has the greater absolute value, and is negative if the negative number has the greater absolute value. The absolute value of the sum is determined by finding the difference between the absolute values of the numbers. This process may be linked to the geometric interpretation of moving in different directions on the number line.

12. Apply the commutative and associative properties of addition to facilitate the combining of several directed numbers.

13. Explore the dual interpretation of " + " and " - " and distinguish between a sign of operation and a sign of direction. Point out that in an expression such as 7 - 2, whether it is translated seven minus two or as the sum of seven and negative two, the result is independent of the interpretation. Guide pupils to understand also that $(-3) + (+2) + (-7)$ may be written $-3 + 2 - 7$; and that, similarly, $(+5x) + (-7x) + (-2x)$ may be written $5x - 7x - 2x$.

14. Establish the validity of the rules for adding directed numbers as enrichment material. Present demonstrations such as the following:

$$
\begin{aligned}
(+7) + (-12) &= (+7) + \left[(-7) + (-5)\right] && \text{substitution} \\
&= \left[(+7) + (-7)\right] + (-5) && \text{associative property} \\
&= 0 + (-5) && \text{additive inverse} \\
&= -5 && \text{additive identity}
\end{aligned}
$$

15. Explore patterns such as the following to help pupils develop rules for addition:

$$
\begin{aligned}
(+4) + (+2) &= +6 \\
(+4) + (+1) &= +5 \\
(+4) + (\ 0) &= +4 \\
(+4) + (-1) &= +3 \\
(+4) + (-2) &= +2, \text{ and so on.}
\end{aligned}
$$

16. Define subtraction, a - b, as determining what number " n " must be added to " b " in order to yield " a ".

$$a - b = n; \quad b + n = a$$

TEACHING SUGGESTIONS (contd)

17. Develop the generalization for subtraction through a consideration of inverse operations, examples on the number line (vector subtraction), solution of equations, and recognition of pattern. Appeal to pupils by means of several experiences.

18. Present the concept that the inverse of the sum of two numbers is the sum of their inverses, before assigning exercises such as the following:

$$(x + 9) - (x - 7)$$

Approach this problem also from the point of view of $(x + 9) - 1\ (x - 7)$.

19. Develop rules for multiplication of directed numbers through the following approaches:

a. Experimental approach, using concrete, physical analogies, such as:

Deposits and withdrawals, with respect to time
Traveling east and west at given rates, with respect to time

b. Study of patterns, using such examples as:

| | |
|---|---|
| $(-2)(+4) = -8$ | $(-2)(0) = 0$ |
| $(-2)(+3) = -6$ | $(-2)(-1) = +2$ |
| $(-2)(+2) = -4$ | $(-2)(-2) = +4$ |
| $(-2)(+1) = -2$ | $(-2)(-3) = +6$ and so on. |

c. Examination of structure to demonstrate that $(-5)(-3) = +15$:

| | | |
|---|---|---|
| $(-5)(0) = 0$ | | multiplicative property of zero |
| $(-5)\left[(+3) + (-3)\right] = 0$ | | additive inverse and substitution |
| $(-5)(+3) + (-5)(-3) = 0$ | | distributive principle |
| $-15 + (-5)(-3) = 0$ | | substitution |
| $\therefore (-5)(-3) = +15$ | | additive inverse |

20. Stress that the product of any number " n " and (-1) is " -n ".

21. Refer to "An Introduction to Sets and the Structure of Algebra," by Krickenberger and Pearson, for enrichment topics and the development of theorems such as:

"If a and b are elements of set S, then a x (-b) = -(a x b)."

"If a and b are elements of set S, then (-a) x (-b) = a x b."

TEACHING SUGGESTIONS (contd)

22. Define division as the inverse of multiplication and develop generalizations for division of directed numbers. The reciprocal of a number may be described as the number whose product with the given number is equal to one.

EVALUATION

See the evaluation section for Unit I, pages 5 and 6 of this guide.

# UNIT V: EQUATIONS, INEQUALITIES, AND PROBLEM SOLVING

## GOALS

Pupils should be able to demonstrate that they have learned to:

1. Transform equations into equivalent equations.
2. Solve open sentences in which the domain of the variable is the directed numbers.
3. Identify the properties of inequality.
4. Solve inequalities and graph solution sets.
5. Analyze and solve problems.

## SEQUENTIAL DEVELOPMENT OF THE UNIT

### Transformation of Equations

The same principles that govern transformations in the set of non-negative numbers are valid with open sentences in the set of directed numbers. (See Transformation of Equations, Unit III.)

The addition and subtraction properties of equality may be extended to include the addition and subtraction of terms containing the variable. Equations whose solution sets contain negative numbers can be solved by employing the same procedures and strategies which were used when roots were limited to the positive numbers and zero.

The properties of equality are applied successively to transform equations into equivalent equations and, finally, to determine the solution when the replacement set includes the directed numbers.

### Properties of Inequality

Inequalities possess many of the properties of equations. They remain true and an equivalent inequality is derived, if the same quantity is added to or subtracted from both members, or if both members are multiplied or divided by the same <u>positive</u> number.

Multiplication or division by negative numbers changes (or reverses) the sense or order of the inequality. The <u>direction</u> (greater than or less than) in which the inequality sign points is the <u>sense</u> of the inequality.

Since $5 > 3$, it follows that:

$5 + 1 > 3 + 1$, $5 - 2 > 3 - 2$, and $5 \times 4 > 3 \times 4$; but $5(-1) < 3(-1)$.

## SEQUENTIAL DEVELOPMENT OF THE UNIT

### Properties of Inequalities (contd)

The graphical solution of an inequality is shown by finding the points on the number line or the region in the plane or in space where the inequality holds true. For example, $x > 5$ has for its solution all points on the line to the right of the point which corresponds to + 5. (It also has for its solution all points in the region to the right of the line whose equation in rectangular coordinates is $x = 5$.) The inequality $x^2 + y^2 + z^2 < 1$ has for its solution all points within the sphere $x^2 + y^2 + z^2 = 1$.

Inequalities are transformed to equivalent inequalities by applications of the properties of inequality, and solution sets are graphed.

### Solution of Problems

The technique of solving problems involves both the careful analysis of the information provided and the organization of facts in a coherent, logical sequence. It is essential that the variable be identified at the outset and that the pupil recognize what he is required to find. It is necessary to translate accurately the conditions imposed upon the variable into a mathematical sentence. That sentence must be solved, and it is vital that the solution be reasonable and that it satisfy the circumstances described by the problem.

A plan for solving problems is evolved, and this scheme is applied to the solution of problems about consecutive integers, angles, uniform motion, and mixtures.

## TEACHING SUGGESTIONS

The teacher may find it helpful to:

1. Review the properties of equality and require pupils to explain why each transformation produces an equivalent equation. However, reasons need be supplied only in the illustrations provided in class and in a few of the assigned problems.
2. Extend the principles developed in Unit III to include open sentences in which the domain of the variable is the set of directed numbers.
3. Point out that the solving of equations containing directed numbers proceeds in the same manner as does the solving of equations with only positive terms and that in these cases the same principles which governed earlier transformations apply.
4. Emphasize that the transformations convert the equation into an <u>eqivalent</u> equation whose solution set can be determined by inspection, thus making the solution obvious. Two open sentences are equivalent if and only if their solution sets are identical.

TEACHING SUGGESTIONS (contd)

5. Assign for additional practice and drill that section in Unit III which involves equations having the variable in both members and the set of word problems which follows. Amplify the addition and subtraction properties, if necessary, to permit the adding and subtracting of terms containing the variable to both members of an equation.

6. Re-examine the sequential development and teaching suggestions presented in Unit III, and review with pupils the strategy for the solving of equations. Point out the need to simplify each member of the equation before applying the properties of equality.

7. Expand the principles which govern transformations to include the adding of polynomials to both members of an equation without altering the solution set.

8. Discuss the addition of the <u>additive inverse</u> as a substitute for applying the subtraction property. Point out, though, that the result is the same, irrespective of the technique used. Many pupils find it easier to recall only one property and to think solely in terms of using the <u>inverse</u> element.

9. Reverse the order in which the principles are applied with an equation such as $3x + 4 = 22$ to show that the same root is obtained if one divides before applying the subtraction (or addition) property. Help pupils to recognize which order of operations is better when solving equations of this kind.

10. Insist that pupils check each apparent root by substituting it for the variable in the <u>original</u> equation. Errors may occur in transforming from one equivalent equation to another. Although the result may be a root of some of the derived equations, it will not satisfy the original equation.

11. Illustrate that for any two real numbers, a and b, a <u>is greater than</u> b, if and only if a - b is positive; and a <u>is less than</u> b, if and only if a - b is negative. Show, too, for each pair of real numbers, a and b, one and only one of the following relations is true: $a < b$, $a = b$, or $a > b$.

12. Refer to the material presented in Unit III on properties of the equivalence relation. The inequality relation is <u>transitive</u>, non-reflexive, and non-symmetric.

13. Prepare pupils to accept solutions as entire sets of numbers, for example, as all directed numbers less than five, rather than as a set containing only one element, as is the case with linear equations.

14. Develop the proofs of the properties of the order relation described in the Teacher's Manual only with mathematically talented pupils who progress more swiftly and meet normal requirements sooner than usual.

15. Stress that multiplying or dividing both members of an inequality by a <u>negative</u> number <u>reverses</u> the <u>sense</u> (or direction) of the inequality. Use the number line, as suggested in the text, to illustrate that the order of the inequality is reversed when both members are multiplied by a negative number.

TEACHING SUGGESTIONS (contd)

16. Transform inequalities into equivalent ones in simpler from so that members of the solution set may readily be determined.

17. Require that pupils graph the solution set of only a few inequalities.

18. Provide for the more able students by considering the section on Pairs of Inequalities and the mathematical interpretation of the conjunctions "and" and "or."

19. Place great emphasis on the need for careful analysis of the facts presented by word problems and for an accurate diagnosis by the pupil of what he is being asked to find. Provide more examples than are included in the book. Anticipate that several problems in the assignment result in inequalities.

20. Encourage pupils to invent several interpretations for an algebraic expression such as $4x + 3$ or for an open sentence to develop a feeling for contexts in which the expression or sentence can arise. Describe familiar situations in the physical world and provide aid in translating these situations into algebraic expressions.

21. Furnish opportunity in class for additional practice in translating the situations described by problems into their algebraic equivalents and into the necessary open sentences. Review the interpretation of phrases such as increased by, less than, and exceeds. Remind pupils about the use of grouping symbols, when necessary, as in "fifteen decreased by the sum of a certain number and nine."

22. Suggest that pupils prepare sketches as an aid to visualizing and summarizing the facts of word problems, particularly about geometric figures in which the relationship often can be seen best by means of a drawing.

23. Permit pupils to employ a chart to collect the information and to orgainze the data, but discourage the use of a chart as a device upon which the pupil becomes so dependent that he cannot complete a problem if he fails to recall the titles of the columns.

24. Assist pupils who encounter difficulties with problems by having them restate problems in their own words. Although pupils should not be directed to try to fit problems into fixed patterns, they may find it helpful to compare a problem with others they have solved in order to detect similarities and differences.

25. Explain that each integer has an immediate successor in the set of integers. Be sure that pupils understand how to represent consecutive odd or consecutive even integers.

TEACHING SUGGESTIONS (contd)

26. Present the material about directed angles only to more able students. For average classes, limit the definitions of complementary and supplementary to pairs of positive angles. Rotation is not considered again until the study of trigonometry.

27. Explain in greater detail how to represent two numbers when their sum is known. This knowledge is required for motion problems in which various times must be defined when only total time is given, and for mixture problems in which various numbers of items must be represented when only the total number sold is given.

28. Review relationships in problems involving distance, rate, and time. Before presenting any problems in which variables are involved, have pupils solve arithmetic problems involving similar situations.

    a. Determine the distance an automobile travels in four hours if it moves at the rate of 35 miles per hour. Develop an understanding of the formula $d = rt$.

    b. Present problems in which pupils must find the rate when distance and time are given or in which they must find the time when distance and rate are reported. In some classes, the one motion formula may be sufficient; in others, you may wish to develop the formulas for rate and time.

    c. Practice solving distance problems involving one object or vehicle. For example, if an airplane averages 580 miles per hour, how far will it go at this speed in three hours?

    d. Solve distance problems involving two objects or people. For example, Rick and Steve, on bicycles, start from the same place at the same time and ride in opposite directions. Rich travels at the rate of 5 mph, and Steve at 8 mph. How far apart are they in one hour? In two hours? In four hours? Encourage pupils to use a diagram or sketch to represent these facts.

    e. Solve problems in which time is the variable. For example, Rick and Steve are 45 miles apart. They start toward each other at the same time. Rick rides at an average speed of 9 miles per hour, and Steve rides at an average speed of 6 miles per hour. In how many hours after they start will they meet? Emphasize that each boy travels the same number of hours.

29. Allow pupils to present alternative solutions to problems. Guide them to recognize that there is more than one way to solve a problem.

TEACHING SUGGESTIONS (contd)

30. Supply material to supplement the section on mixture problems. Point out that the seller nets the <u>same</u> amount of money after blending or mixing the ingredients as he would have received had he not mixed them.

31. Stress the importance of checking results with the requirements and conditions stated in the problem.

EVALUATION

See the evaluation section for Unit I, pages 5 and 6 of this guide.

# UNIT VI: WORKING WITH POLYNOMIALS

## GOALS

Pupils should be able to demonstrate that they have learned to:

1. Add and subtract polynomials.
2. Solve equations in which symbols of inclusion are used to indicate addition and subtraction of polynomials.
3. Multiply polynomials
4. Simplify expressions and solve equations in which grouping symbols are used.
5. Find the solutions of problems about areas.
6. Determine the powers of polynomials.
7. Divide polynomials.

## SEQUENTIAL DEVELOPMENT OF THE UNIT

### Addition and Subtraction of Polynomials

A polynomial is an algebraic expression consisting either of one term or of the sum of terms, such that each term either is a numeral or is the indicated product of a numeral and a positive integral power of one or more variables.

A polynomial has been defined in terms of a variable, with the set of rational numbers as its domain. Hence, operations with polynomials possess the same properties as do operations with rational numbers. If two or more terms have the same variables, and each variable, respectively, is used as a factor the same number of times, the terms are said to be <u>like</u> (or similar) <u>terms</u>.

Expressions are simplified and polynomials are added by using the commutative, associative, and distributive properties to combine similar terms. Subtraction of a polynomial is performed by the addition of its additive inverse. Equations are solved in which parentheses are used to indicate addition and subtraction of polynomials.

### Multiplication of Polynomials

If a number, called the base, is used a given number of times as a factor, the <u>product</u> is called the <u>power</u> of that base. Any product in which all the factors are identical is a <u>power</u>; the common factor is the base, and the number of such factors is the exponent. Thus, $4^3 = 4 \times 4 \times 4 = \underline{64}$. Then 64 is the third <u>power</u> of the base 4.

The laws of exponents are developed and are used with the commutative and associative properties to determine coefficients and variable factors of monomial products. Expressions are simplified by performing the indicated multiplications and combining similar terms.

## SEQUENTIAL DEVELOPMENT OF THE UNIT

### Multiplication of Polynomials (contd)

To multiply a polynomial by a monomial, the distributive property is applied. An equivalent expression is found by multiplying each term of the polynomial by the monomial factor. Equations are solved which contain products of polynomials and monomials.

The product of two polynomials is determined by applying the distributive property, multiplying each term of one polynomial by every term of the other, and by combining the similar monomial products. Problems about areas are solved, the powers of polynomials are found, and word problems are solved and checked.

### Division of Polynomials

To divide a monomial by a monomial, apply the rules governing the division of directed numbers, divide the absolute value of the numerical coefficient of the dividend by the absolute value of the numerical coefficient of the divisor, and use the laws of exponents for division to determine the variable factors of the quotient.

If x is a nonzero number, the value of $x^0$ is defined to be 1. If $x \neq 0$, $x^0$ is regarded as the result of subtracting exponents when dividing a quantity by itself:

$$\frac{x^4}{x^4} = x^0 = 1.$$

The distributive property and the fact that division is accomplished by multiplying by the reciprocal of the divisor are used to explain the procedure for finding the quotient of a polynomial and a monomial. Each term of the polynomial is divided by the monomial.

To divide one polynomial by another, arrange the terms of the dividend and divisor in descending powers of one of the variables and follow a sequence of steps similar to the algorithm for arithmetic division.

## TEACHING SUGGESTIONS

The teacher may find it helpful to:

1. Explain that every polynomial is either a monomial or a sum of monomials. If a polynomial is a sum of two monomials, it is called a binomial; if it is the sum of three monomials, it is called a trinomial. The degree of a polynomial with respect to a certain variable is the highest power of that variable appearing in the polynomial.

2. Point out that when a term has a negative coefficient, a plus sign is not written to indicate addition.

   Thus, instead of writing $3x^2 + (-2)x + (-6)$, we write $3x^2 - 2x - 6$.

TEACHING SUGGESTIONS (contd)

3. Discuss with pupils the desirability of arranging the terms in some definite order when operating with polynomials. Using the associative and commutative properties, one may rearrange the terms $3x^2 + 5x^4 - 2x^3 + 6 + x$ in any order. It is customary and convenient to arrange them in either the descending or the ascending order of the powers of the variable. In the text, reference is made to an arrangement of the terms in order of either the decreasing or increasing degree in a particular variable.

4. Review the use of the distributive property to simplify expressions and to illustrate that this application provides a procedure for adding monomials. For example:

$$8xy^2 + 17xy^2 = (8 + 17)xy^2 = 25xy^2$$

5. Establish the validity of the procedure for addingpolynomials by citing the use of the commutative, associative, and distributive properties. For example:

$$(4x^3 - 6x^2 + x + 1) + (-2x^3 - 5x^2 + 2x - 2)$$

$$= (4x^3 - 2x^3) + (-6x^2 - 5x^2) + (x + 2x) + (1 - 2)$$ associative and commutative properties

$$= (4 - 2)x^3 + (-6 - 5)x^2 + (1 + 2)x + (1 - 2)$$ distributive property

$$= 2x^3 - 11x^2 + 3x - 1.$$

A simpler and equivalent expression is thereby obtained. It is not intended, however, that pupils repeat this process each time they do an exercise. This can be made part of the teacher's demonstration, but pupils should be encouraged to use a more efficient technique and one with which they can be more comfortable.

6. Illustrate the vertical arrangement for addition and use the commutative and associative properties to explain the alignment of similar terms underneath one another.

7. Require that pupils check several addition problems by assigning numerical values to the variables and evaluating the expressions. Discuss the disadvantages of selecting 0 or 1 as replacements.

8. Define the additive inverse of a polynomial as another polynomial, each of whose terms is the additive inverse of the corresponding term of the original polynomial: $3x^3 - 5x + 2$ and $-3x^2 + 5x - 2$ are additive inverses. Provide practice in finding additive inverses of polynomials.

TEACHING SUGGESTIONS (contd)

9. Indicate that subtraction of polynomials parallels the meaning of subtraction of directed numbers and that subtracting a polynomial is accomplished by adding its <u>additive inverse</u>.

10. Insert the following step, explicitly showing the use of the definition of subtraction:

$$6x^2 + 5x - 4 - (9x^2 + 7x - 3) = 6x^2 + 5x - 4 + \boxed{-(9x^2 + 7x - 3)}$$

before progressing to $6x^2 + 5x - 4 - 9x^2 - 7x + 3$.

The pupils need not repeat this when they do the exercises. However, it should prevent their thinking solely in terms of the mechanical removal of parentheses. You may wish also to interpret this solution in terms of multiplication by -1:

$$6x^2 + 5x - 4 - 1(9x^2 + 7x - 3).$$

11. Supplement the textbook exercises on subtracting polynomials. Suggest solutions in both the horizontal and vertical forms. Encourage checking both by substituting numerical values for the variables in the minuend, subtrahend, and difference, and by adding the difference and the subtrahend to see whether their sum is the same as the minuend.

12. Include a more detailed analysis of the solution of equations in which parentheses appear to indicate addition and subtraction. Explain that first the indicated operations are performed, and then the properties are applied to produce equivalent equations.

13. Emphasize that the laws of exponents apply only when the bases of the powers are the same. Develop the law of exponents for multiplication by showing that $a^m \cdot a^n = a^{m+n}$. Use numerical illustrations and the associative property to demonstrate that

$$x^4 \cdot x^3 = (x \cdot x \cdot x \cdot x) \cdot (x \cdot x \cdot x) = x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x = x^7$$

14. Stress that $x^4 \cdot x^3 \neq x^{12}$, using numerical illustrations. Stress also that $2^4 \cdot 2^3 \neq 4^7$.

15. Have pupils employ the rule of exponents for multiplication, together with the commutative and associative properties, to express the product of monomials:

$$(-4x^2y^2)(-5xy^3) = (-4)\overset{(-5)}{(}x^2 \cdot x)(y^2 \cdot y^3)$$

16. Examine the difference between expressions such as $3x^2$ and $(3x)^2$; and have pupils note that, in raising a product to a power, each of the factors of the product is raised to that power.

TEACHING SUGGESTIONS (contd)

17. Develop the law $(ab)^m = a^m b^m$ by showing that

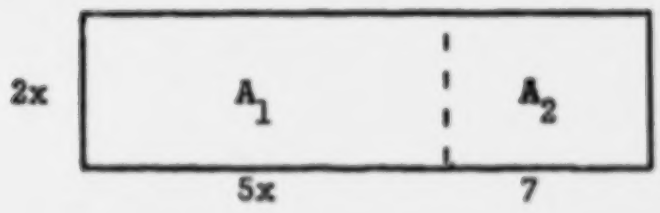
$$1000 = 10^3 = (2 \cdot 5)^3 = 2^3 \cdot 5^3 = 8 \cdot 125 = 1000.$$

18. Inspect exercises such as $(-7x^3)^2$, and use the laws and the properties to interpret the example to mean:

$$(-7x^3)(-7x^3) \quad \text{or} \quad (-7)^2(x^3)^2$$
$$= (-7)(-7)(x^3)(x^3) = 49x^6$$

19. Place special emphasis on the distinction between $a^{mn}$ and $a^{m+n}$.

20. Employ the distributive property and consider geometric examples to interpret the product of 2x and (5x + 7):




The area of the large rectangle equals the sum of the areas of rectangles $A_1$ and $A_2$.

21. Provide additional practice in solving equations, such as:

$$7(5x - 3) - 4(3x - 2) = 10$$

22. Assign exercises in which solutions are reported as polynomials. For example, suppose that a man drove a car for five hours at (x + 30) miles per hour, and for six hours at (x + 45) miles per hour. Represent the total distance traveled.

23. Accompany the demonstrations of multiplication of a polynomial by a polynomial with geometric interpretations and with numerical illustrtions, so that pupils may see the correspondence.

24. Point out the repeated use of the distributive property of multiplication relative to addition in finding the product of two polynomials.

25. Instruct pupils to draw sketches to represent the situations described in the problems about areas and in the set of exercises following "Powers of Polynomials." Pupils often have considerable difficulty with these assignments. You may choose to select only a few of these exercises for this time and to assign other later in the course.

26. Show that raising to a power is not distributive relative to addition and subtraction; that is, that $(a + b)^2 \neq a^2 + b^2$. Use numerical and geometric illustrations to assure that pupils recognize that this solution is not correct.

TEACHING SUGGESTIONS (contd)

27. Stress the property of quotients, and illustrate that:

$$\frac{51 \cdot 12}{3} \neq \frac{51}{3} \cdot \frac{12}{3}$$

28. Use the multiplicative property of 1 to derive the rules of exponents for division:

$$\frac{28}{4} = \frac{7 \cdot 4}{4} = 7 \cdot \frac{4}{4} = 7 \cdot 1 = 7$$

$$\frac{x^8}{x^3} = \frac{x^5 \cdot x^3}{x^3} = x^5 \cdot \frac{x^3}{x^3} = x^5 \cdot 1 = x^5 = x^{8-3}$$

29. Develop the concept of zero exponents: $1 = 4^3 \div 4^3 = 4^{3-3} = 4^0 = 1$. Show the consistency of the laws, and indicate that, although four used as a factor zero times is meaningless, it is defined as one to preserve the uniformity of the law. Furthermore,

$$\text{if } a^m \cdot a^0 = a^{m+0} = a^m,$$

$$\text{then } a^0 = \frac{a^m}{a^m} = 1.$$

30. Introduce division of a polynomial by a monomial as a joint application of the distributive property and the concept of a multiplicative inverse.

31. Present examples of division of integers to accompany the illustration of division of a polynomial by a polynomial.

32. Place restrictions on the divisor. Restrict from the replacement set all numbers that make either the denominator or the divisor zero.

EVALUATION

See the evaluation section for Unit I, pages 5 and 6 of this guide.

# UNIT VII: SPECIAL PRODUCTS AND FACTORING

## GOALS

Pupils should be able to demonstrate that they have learned to:

1. Identify prime numbers and greatest common monomial factors.
2. Multiply the sum and difference of two numbers.
3. Factor the difference of two squares.
4. Square binomials.
5. Factor trinomial squares.
6. Multiply binomials at sight.
7. Factor quadratic trinomials.
8. Factor completely.
9. Solve quadratic equations by factoring.
10. Use factoring in problem solving.

## SEQUENTIAL DEVELOPMENT OF THE UNIT

### Greatest Common Monomial Factor

When two or more expressions are multiplied, each is a <u>factor</u> of the product. 2 and 3 are factors of 6. In a special sense, $\frac{1}{13}$ and 78 are factors of six, and $-14$ and $-\frac{3}{7}$ are factors of six; therefore it is apparent that if fractions are admitted as factors, there is no limit to the number of factors of 6. For this reason, there is need for agreement as to what sets of numbers are to be considered in indentifying factors.

In this unit, the factors of integers will be limited to a specified subset of the set of integers (the prime numbers). The <u>factor</u> of a polynomial whose numerical coefficients are integers, then, will be one of two or more polynomials with integral coefficients whose product is the given polynomial.

An expression which has <u>exactly two different</u> factors (itself and one) is said to be <u>prime</u>. Any integer can be expressed as a product of one and only one set of primes. The instruction, "to factor," means to write the expression as the <u>indicated product</u> of its prime factors.

While $5x + 5y$ may be written as $10(\frac{1}{2}x + \frac{1}{2}y)$ or as $\frac{5}{4}(4x + 4y)$, we are concerned here with indicated products involving only polynomials in which all coefficients are integers. The most useful form of $5x + 5y$ as an indicated product is $5(x + y)$ in which 5 and $(x + y)$ are the factors of $5x + 5y$ and 5 is the <u>common factor</u> of $5x$ and $5y$.

## SEQUENTIAL DEVELOPMENT OF THE UNIT

### Greatest Common Monomial Factor (contd)

Integers are factored, the greatest common factors of pairs of integers are determined, and the greatest common monomial factors of polynomials are found.

### Special Products

The product of the sum and difference of two expressions is the difference between the square of the first expression and the square of the second.

The difference of the squares of two quantities is the product of the sum and difference of the quantities.

The square of a binomial is found by summing the square of the first term of the binomial, twice the product of the two terms, and the square of the second term.

A trinomial is a perfect square if (1) two terms (usually, the first and last terms) are squares of monomials, and (2) the other term is twice the product of these monomilas (or, this term is twice the product of the square roots of the other two terms).

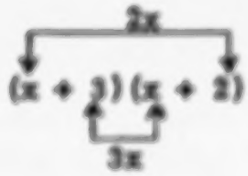
The products of the sums and differences of quantities are found, and binomials are squared. The differences of two squares and trinomial squares are factored.

### Quadratic Trinomials

Application of the distributive peoperty permits the multiplication of two binomials mentally:

$$(x + 3)(x + 2) = (x + 3)(x) + (x + 3)(2)$$
$$= (x^2 + 3x) + (2x + 6)$$
$$x^2 + 5x + 6$$

The product, $x^2 + 5x + 6$, is the sum of (1) the product of the first terms of the binomials, (2) the sum of the two products identified in the sketch below, and (3) the product of the last terms of the binomials.




A quadratic term is a term of the second degree. A quadratic polynomial is one in which the term of the highest degree is the second power (or square) of the variable.

## SEQUENTIAL DEVELOPMENT OF THE UNIT

### Quadratic Trinomials (contd)

In factoring trinomials, if the numerical coefficient of the second-degree term is 1, it is necessary to find two numbers, when their product and sum are given.

Example: Find two integers whose product is -54 and whose sum is 15.

Solution: $54 = 1 \times 54 = 2 \times 27 = 3 \times 18 = 6 \times 9$
$-3$ and 18 are the numbers; $18 + (-3) = 15$.

Binomials are multiplied at sight, trinomials are factored, the general method of factoring quadratic trinomials is considered, and complete factoring is accomplished.

### Quadratic Equations

Second-degree polynomial equations are commonly called quadratic equations. The standard (or general) form of the quadratic equation is $ax^2 + bx + c = 0$, in which $a \neq 0$.

For any two expressions, a and b, $ab = 0$, if and only if $a = 0$ or $b = 0$; that is, if at least one of the factors is zero.

To solve a quadratic equation by factoring, (1) transform the equation into an equivalent equation which appears in the standard form, (2) separate the left member into its factors, (3) set each factor equal to zero, (4) solve the two resulting linear equations, and (5) check each root in the original equation.

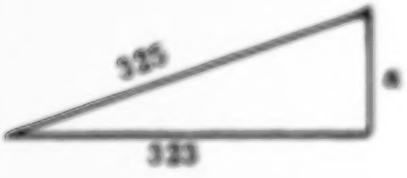
Quadratic equations are solved by factoring, and factoring is used in problem solving.

## TEACHING SUGGESTIONS

The teacher may find it helpful to:

1. Establish a need for factoring with illustrations of the advantage of factoring in problem solving and of the short cuts in computation which factoring provides.

   Consider the right triangle:




$a^2 + (323)^2 = (325)^2$

$a^2 = (325)^2 - (323)^2$

Using a technique which pupils will learn in this unit, this may be rewritten:

This procedure was considerably more rapid than having to square the two numbers and then subtract.

$a^2 = (325 + 323)(325 - 323)$

$a^2 = (648)(2)$

$a^2 = 1296$ and $a = 36$

TEACHING SUGGESTIONS (contd)

2. Account for the limitations placed on the sets of numbers from which factors may be selected. For example, $(2)(10\frac{1}{2})$, $(3)(7)$, $(5)(4.2)$, $(2.4)(8.75)$ all have the value 21, but the fact that $3 \cdot 7 = 21$ is the result which is generally most helpful. A number such as 23 is prime because it has exactly two different integral factors, itself and one. However, if fractions are permitted, 23 has unlimited pairs of factors.

   (46 and $\frac{1}{2}$), (69 and $\frac{1}{3}$), and so on.

3. Verify that, except for sign and order, every integer can be uniquely factored into prime factors.

4. Develop a systematic approach to decomposing a number into its factors. For example, to find the prime factors of 420:

   $2 \mid 420$
   $2 \mid 210$
   $3 \mid 105$
   $5 \mid 35$
   $7$

   $420 = 2^2 \cdot 3 \cdot 5 \cdot 7$

   Start with the lowest prime and continue to divide with it as long as integral quotients result. When this fails, go to the next prime, and so on, until the ultimate quotient is itself a prime.

5. Supply arithmetic examples of monomial factors and caution pupils about the need of identifying such factors as a first step in all factoring operations.

   $246 = 200 + 40 + 6$
   $= (2 \times 100) + (2 \times 20) + (2 \times 3)$
   $= 2 \times (100 + 20 + 3) = 2 \times 123$

6. Emphasize the use of the distributive property throughout this unit. Illustrate that, when a polynomial is expressed as the product of factors, the two expressions are equivalent.

   $7a^2x - 21ax^2 = 7ax(a - 3x)$ is true for all replacements of a and x.

7. Review greatest common factor. Show how to determine the greatest common monomial factor:

   $6x^2y + 15xy^2$ — The greatest common factor of 6 and 15 is 3; the gcf of $x^2y$ and $xy^2$ is $xy$; hence, $6x^2y + 15xy^2 = (3xy)(2x + 5y)$.

   $18a^3b^4 - 12a^2b^3$ — The greatest common factor of 18 and 12 is 6; the gcf of $a^3b^4$ and $a^2b^3$ is $a^2b^3$; hence, $18a^3b^4 - 12a^2b^3 = 6a^2b^3(3ab - 2)$.

TEACHING SUGGESTIONS (contd)

8. Restrict instruction in the notion of factoring by grouping to only the more able pupils. If developed more slowly than in the text and approached in logical sequence, this technique can prove most useful. After considerable practice with items such as
$x(a + b) - y(a + b) = (x - y)(a + b)$ and
$2a(x - y) + 3b(x - y) = (2a + 3b)(x - y)$, instruction may progress to situations where appropriate grouping is necessary:

$$\begin{aligned} x^2 - ax + bx - ab &= (x^2 - ax) + (bx - ab) \\ &= x(x - a) + b(x - a) \\ &= (x + b)(x - a) \end{aligned}$$

If this point of view is pursued, it later can be shown that:

$$\begin{aligned} x^2 + 7x + 12 &= x^2 + 3x + 4x + 12 \\ &= x(x + 3) + 4(x + 3) \\ &= (x + 4)(x + 3) \end{aligned}$$

$$\begin{aligned} 15x^2 - x - 6 &= 15x^2 - 10x + 9x - 6 \\ &= 5x(3x - 2) + 3(3x - 2) \\ &= (5x + 3)(3x - 2) \end{aligned}$$

9. Show the usefulness of detecting greatest common monomial factor in problems and computational exercises, such as:

   a. A cylindrical iron pipe has an inside diameter of 6 inches and an outside diameter of 8 inches. Find the area of the cross-sectional ring.

   $A = A_1 - A_2 = \pi 4^2 - \pi 3^2 = \pi(4^2 - 3^2) = 7\pi$

   b. $64 \cdot 256 + 36 \cdot 256 = 256(64 + 36) = 256(100)$.

   c. 4% of \$2500 + 4% of \$1500 = $.04(2500 + 1500)$.

10. Point out that, if the average of two numbers is a multiple of 10, it is easy to find their product mentally by using the special product of the sum and difference of two numbers.

    For example, the average of 18 and 22 is 20. 18 is two less than 20, and 22 is two more than 20. Thus, 18 x 22 can be written $(20 - 2)(20 + 2)$.

    $36 \times 44 = (40 - 4)(40 + 4) = 40^2 - 4^2 = 1600 - 16 = 1584$.

    Many products can, by inspection also, be changed to a product of the sum and difference of two numbers.

    For example, $2\frac{1}{8} \times 1\frac{7}{8}$ is easily recognized as $(2 + \frac{1}{8})(2 - \frac{1}{8})$.

    $5\frac{1}{2} \times 4\frac{1}{2} = (5 + \frac{1}{2})(5 - \frac{1}{2}) = 5^2 - (\frac{1}{2})^2 = 25 - \frac{1}{4} = 24\frac{3}{4}$.

TEACHING SUGGESTIONS (contd)

11. Elaborate on how to recognize a monomial square and to factor the difference of two squares. For example:

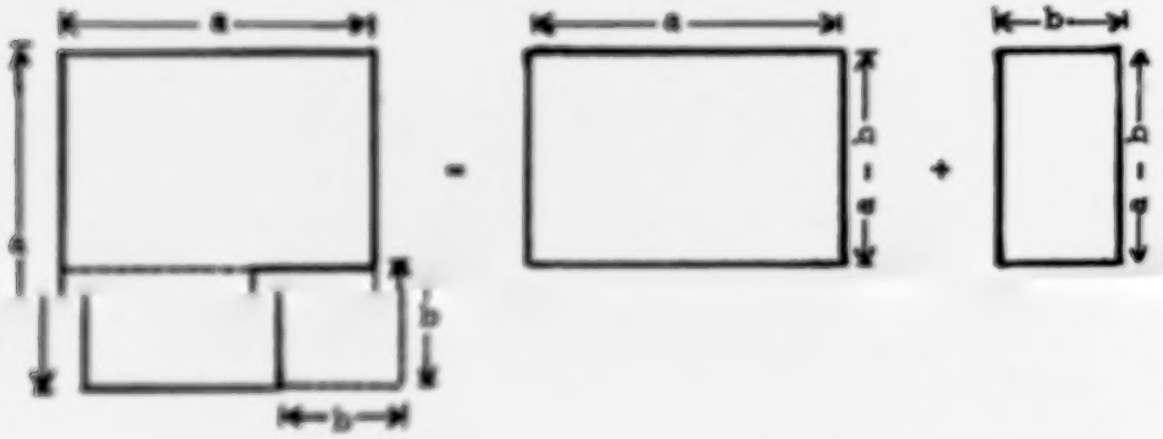
$$9a^2 - 16x^2y^4 = (3a)^2 - (4xy^2)^2$$
$$= (3a + 4xy^2)(3a - 4xy^2)$$

12. Encourage pupils to look first for a common factor. Very often it disguises the difference of two squares. For example:

$$2x^3 - 32x = 2x(x^2 - 16) = 2x(x + 4)(x - 4)$$

13. Use geometric illustrations to demonstrate the square of (a + b) and the square of (a - b) and to develop other algebraic identities.

Factoring, too, may be illustrated by this method:




$$a^2 - b^2 = a(a - b) + b(a - b)$$
$$a^2 - b^2 = (a - b)(a + b)$$

14. Suggest that the test which is described under sequential development may be used for discovering when a trinomial is a square of a binomial. Distinguish between the square of a sum and the square of a difference by examination of the middle term of the trinomial square.

Example: $25k^2 + 35k + 49$ Although $25k^2 = (5k)^2$ and $49 = (7)^2$, 35k is <u>not</u> equal to twice the product of (5k) and (7). Hence, $25k^2 + 35k + 49$ is <u>not</u> a perfect trinomial square.

15. Omit, at this time, reference to negative square roots, such as the following:

$$25x^2 = (-5x)^2.$$

TEACHING SUGGESTIONS (contd)

16. Anticipate the factoring assignments which follow by discussing the relationship among the coeffcients of terms in the product and the coefficients of terms in the binomial factors when multiplying mentally.

17. Regard "mixed numbers" as binomials, and find the product of two mixed numbers, as follows:

$$7\tfrac{1}{2} \times 9\tfrac{1}{4} = (7 + \tfrac{1}{2})(9 + \tfrac{1}{4})$$

$$= (7 \times 9) + (7 \times \tfrac{1}{4}) + (\tfrac{1}{2} \times 9) + (\tfrac{1}{2} \times \tfrac{1}{4})$$

$$= 63 + 1\tfrac{3}{4} + 4\tfrac{1}{2} + \tfrac{1}{8} = 69\tfrac{3}{8}$$

Geometric interpretation of this problem will help to forestall manipulation errors such as "$7\tfrac{1}{2} \times 9\tfrac{1}{4} = 63\tfrac{1}{8}$."

18. Examine the clues supplied by the signs when factoring a trinomial product, such as $ax^2 + bx + c$ in which $a > 0$.

a. If b and c are positive, both binomial factors are sums: $(x + \ )(x + \ )$.

b. If b is negative and c is positive, both binomial factors are differences: $(x - \ )(x - \ )$.

c. If c is negative, the binomial factors are a sum and a difference: $(x + \ )(x - \ )$.

19. Affirm the fact that the given expression and the factored expression are equivalent, as in the following exercises:

a. Factor $x^2 - 3x - 28$

$$x^2 - 3x - 28 = (x - 7)(x + 4)$$

b. Evaluate the expression, $x^2 - 3x - 28$, when $x = 17$.

Most pupils will substitute in the given expression and will encounter difficulty with the computations. By substituting 17 in the factored form, the value can very rapidly be found mentally. Repeat the demonstration. letting $x = 107$, 96, -14, and 1007.

20. Stress that "to factor completely" means to determine <u>all</u> factors. Remind pupils first to seek a common monomial factor. Supplement the section in the text with additional exercises on factoring completely.

TEACHING SUGGESTIONS (contd)

21. Apply the principle, "For real numbers a and b, $ab = 0$ if and only if $a = 0$ or $b = 0$," to solve quiratic equations by factoring. This principle declares that a given equation, such as

$$(x - 3)(x + 2) = 0$$

and the sentence

$$x - 3 = 0 \text{ or } x + 2 = 0$$

have the same solution set $(3, -2)$. The roots are 3 and -2. Pupils often ask, "How can ' x ' stand for two numbers at the same time?" In the case of the open sentence $x^2 - x - 6 = 0$, you can show that 3 and -2 are numbers which, when used as replacements for $x$, make the sentence true.

22. Consider the following situations when solving polynomial equations by factoring:

a. If the polynomial contains a greatest common monomial factor, apply the division peoperty of equality and divide every term in both members of the equation by the greatest common monomial factor.

b. If the polynomial is a perfect square, a multiple (or double) root will be discovered.

c. If the polynomial is multiplied by zero, extraneous roots may be introduced.

d. If the plynomial is divided by the variable, a root may be lost.

23. Postpone consideration of cubic equations for all but the more able pupils.

24. Emphasize that setting factors of a number of a quadratic equation equal to zero is dependent upon having the original equation in standard form. The principle does not apply directly to an equation such as $(2x + 1)(3x - 7) = 3$. This equation first needs to be changed to $6x^2 - 11x - 7 = 3$; then, to $6x^2 - 11x - 10 = 0$.

TEACHING SUGGESTIONS (cont'd)

35. Illustrate the solution of word problems leading to quadratic equations with an exercise, such as the following:

Find two consecutive positive integers the sum of whose squares is 181.

the first integer = x

the next consecutive integer = x + 1

$$x^2 + (x+1)^2 = 181$$
$$x^2 + x^2 + 2x - 180 = 0$$
$$2x^2 + 2x - 180 = 0$$
$$x^2 + x - 90 = 0$$
$$(x-9)(x+10) = 0$$
$$x - 9 = 0 \qquad x + 10 = 0$$
$$x = 9 \qquad x = -10$$

If $x = 9$, then $x + 1 = 10$.
If $x = -10$, then $x + 1 = -9$.

Since the problem asked for consecutive positive integers, the pair, -10 and -9, are rejected. Remind pupils to check each root with the conditions described by the problem.

36. Demonstrate with more able pupils the following method of factoring a trinomial:

$6x^2 + 17x - 14$

$6(-14) = -84$

$+21 \qquad -4$

two factors of -84 whose sum is +17.

$$6x^2 + (21 - 4)x - 14$$
$$6x^2 + 21x - 4x - 14$$
$$3x(2x + 7) - 2(2x + 7)$$
$$(3x - 2)(2x + 7)$$

EVALUATION

See the evaluation section for Unit I, pages 5 and 6 of this guide.

# A L G E B R A  1

(Alternate Sequence)

Text: Keedy, Jameson, Johnson: Exploring Modern Mathematics, Book 3, Elementary Algebra

## TO THE TEACHER

The text Keedy, Jameson, Johnson: Exploring Modern Mathematics, Book 3, Elementary Algebra is recommended for use with high-ability groups to culminate the on-going sequence presented in Books 1 and 2, by the same authors.

Chapter 7 (Solving Equations) and Chapter 8 (Polynomials) in Book 2 must be completed before pupils advance to Book 3.

The first chapter of Book 3 is essentially the same as the last chapter (Chapter 9, Applied Problems, Conjunctions of Equations) of Book 2. This provides for more flexibility in the use of the series of textbooks.

If pupils have not studied Chapter 9, Book 2, use of the outline which follows is appropriate.

If pupils have completed Chapter 9, Book 2, the following sequence may be altered to include the study of Chapter 5 during the first semester. Chapter 10 and/or portions of Chapter 7 may be added to Chapters 6, 8, and 9 to complete the second semester's assignment.

Careful consideration should be given to the commentary for teachers, teaching hints, and overviews of each chapter presented in the introduction and to the suggestions included for each chapter in the Teacher's Edition of the text.

The teacher may find it helpful to employ some of the following additional suggestions.

## UNIT 1: APPLIED PROBLEMS, CONJUNCTIONS OF EQUATIONS
(33 Teaching Days)

Develop a technique for solving problems which places greater emphasis on identifying the variable, representing it algebraically, and translating accurately the conditions imposed upon the variable into a mathematical sentence. (See "Solution of Problems," page 24 of the Instructional Guide, and Teaching Suggestions 19-29, pages 26 and 27.)

Assign only a few applied problems at one time, and include one or two at regular intervals throughout the semester.

Solve pairs of equations graphically so that pupils may understand more clearly the conditions under which the equations are inconsistent (the lines are parallel), the equations are dependent (the lines are coincident), and the equations are independent (the lines intersect).

## UNIT II: SIMILAR FIGURES
(20 Teaching Days)

Employ practice exercises on pages 16 and 21 for review if Chapter 9, Book 3, was completed in the eighth Grade and you are beginning with this chapter.

Explore some examples to illustrate inverse variation in an effort to help pupils recognize and distinguish more clearly the meaning of direct variation.

Include material on Pythagorean property of right triangles; and, even if the new proof is not presented, accomplish the exercises on page 98.

## UNIT III: POLYNOMIALS IN SEVERAL VARIABLES
(17 Teaching Days)

Extend the system of polynomials in one variable with rational number coefficients to include polynomials in several variables.

Examine the development and suggestions for Unit VI, "Working with Polynomials," pages 29-34 of the Instructional Guide, for amplification of topics such as similar terms, addition of polynomials, additive inverses and subtraction, and multiplication.

Consult the Instructional Guide, Unit VII, "Special Products and Factoring," pages 35-43, for further development of complete factoring and an alternate method of factoring trinomials.

Encourage pupils to prepare sketches as an aid to visualizing and summarizing the facts of applied problems.

Select applied problems describing physical situations with which pupils are more familiar.

## UNIT IV: NUMBER SENTENCES AND PROOFS
(20 Teaching Days)

Stress the need, when preparing proofs, for supplying a reason to support each assertion that is made as one progresses toward the conclusion.

Anticipate that pupils may encounter difficulty devising formal proofs; recognize that proficiency will come only after considerable practice.

Refer to the Instructional Guide, Unit V, "Equations, Inequalities, and Problem Solving," page 24, Teaching Suggestion No. 4, for comment on equivalent sentences and to pages 23 and 24 for additional information on properties of inequalities.

Illustrate the division of one polynomial by another by arranging the terms of the dividend and divisor in descending powers of one of the variables and following a sequence of steps similar to the algorithm for arithmetic division.

If Chapter 9, Book 2, has been completed previously, and if the current semester's work commenced with Chapter 2, Book 3, then Chapter 5 should be included in Algebra 1, as follows:

## UNIT V: THE SYSTEM OF REAL NUMBERS
(25 Teaching Days)

Expand the number system to include the irrational numbers, thus completing the set of real numbers.

Clarify the meaning of absolute value by referring to the Instructional Guide, Unit IV, "The Negative Numbers," page 17, and to Teaching Suggestions Nos. 3 and 9, page 19.

Consult a source such as "An Introduction to Sets and the Structure of Algebra" by Erickenberger and Pearson for further development of theorems.

Spend considerable time developing the section on polynomials with real coefficients. This material provides a background and foundation vital for an understanding of many of the concepts considered in Chapters Eight and Nine.

Emphasize that every point on the plane corresponds to an ordered pair of numbers